**Danube Mathematical Competition 2012**

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by parmenides51, Ghd

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| – | Junior |
| 1 | a) Exist $a, b, c, \in N$, such that the numbers $ab+1, bc+1$ and $ca+1$ are simultaneously even perfect squares ?<br>b) Show that there is an infinity of natural numbers (distinct two by two) $a, b, c$ and $d$, so that the numbers $ab+1, bc+1, cd+1$ and $da+1$ are simultaneously perfect squares. |
| 2 | Consider the natural number prime $p, p>5$. From the decimal number $\frac{1}{p}$, randomly remove $2012$ numbers, after the comma. Show that the remaining number can be represented as $\frac{a}{b}$ , where $a$ and $b$ are coprime numbers , and $b$ is multiple of $p$. |
| 3 | Let $ABC$ be a triangle with $\angle BAC = 90^o$. Angle bisector of the $\angle CBA$ intersects the segment $(AB)$ at point $E$. If there exists $D \in (CE)$ so that $\angle DAC = \angle BDE = x^o$ , calculate $x$. |
| 4 | Let $A$ be a subset with seven elements of the set $\{1, 2, 3, ..., 26\}$.<br>Show that there are two distinct elements of $A$, having the same sum of their elements. |
| – | Senior |
| 1 | Given a positive integer $n$, determine the maximum number of lattice points in the plane a square of side length $n + \frac{1}{2n+1}$ may cover. |
| 2 | Let $ABC$ be an acute triangle and let $A_1$, $B_1$, $C_1$ be points on the sides $BC, CA$ and $AB$, respectively. Show that the triangles $ABC$ and $A_1B_1C_1$ are similar ($\angle A = \angle A_1, \angle B = \angle B_1, \angle C = \angle C_1$) if and only if the orthocentre of the triangle $A_1B_1C_1$ and the circumcentre of the triangle $ABC$ coincide. |
| 3 | Let $p$ and $q, p<q$, be two primes such that $1+p+p^2+...+p^m$ is a power of $q$ for some positive integer $m$, and $1+q+q^2+...+q^n$ is a power of $p$ for some positive integer $n$. Show that $p=2$ and $q=2^t-1$ where $t$ is prime. |
| 4 | Given a positive integer $n$, show that the set $\{1, 2, ..., n\}$ can be partitioned into $m$ sets, each with the same sum, if and only if m is a divisor of $\frac{n(n+1)}{2}$ which does not exceed $\frac{n+1}{2}$. |