Danube Mathematical Competition 2007

www.artofproblemsolving.com/community/c910580

by freemind

1 Let $n \geq 2$ be a positive integer and denote by S_n the set of all permutations of the set $\{1, 2, \ldots, n\}$. For $\sigma \in S_n$ define $l(\sigma)$ to be $\min\limits_{1 \leq i \leq n-1} |\sigma(i+1) - \sigma(i)|$. Determine $\max\limits_{\sigma \in S_n} l(\sigma)$.

---

2 Let $ABCD$ be an inscribed quadrilateral and let E be the midpoint of the diagonal BD. Let $\Gamma_1, \Gamma_2, \Gamma_3, \Gamma_4$ be the circumcircles of triangles AEB, BEC, CED and DEA respectively. Prove that if Γ_4 is tangent to the line CD, then $\Gamma_1, \Gamma_2, \Gamma_3$ are tangent to the lines BC, AB, AD respectively.

---

3 For each positive integer n, define $f(n)$ as the exponent of the 2 in the decomposition in prime factors of the number $n!$. Prove that the equation $n - f(n) = a$ has infinitely many solutions for any positive integer a.

---

4 Let a, n be positive integers such that $a \geq (n-1)!$. Prove that there exist n *distinct* prime numbers $p_1, \ldots, p_n$ so that $p_i | a + i$, for all $i = \overline{1, \ldots, n}$.
