**Danube Mathematical Competition 2008**

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**1** $x, y, z, t \in \mathbb{R}_+^*$:

$$(xy)^{1/2} + (yz)^{1/2} + (zt)^{1/2} + (tx)^{1/2} + (xz)^{1/2} + (yt)^{1/2} \geq 3(xyz + xyt + xzt + yzt)^{\frac{1}{3}}$$

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**2** In a triangle $ABC$ let $A_1$ be the midpoint of side $BC$. Draw circles with centers $A, A1$ and radii $AA_1, BC$ respectively and let $A'A''$ be their common chord. Similarly denote the segments $B'B''$ and $C'C''$. Show that lines $A'A''$, $B'B'''$ and $C'C''$ are concurrent.

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**3** On a semicircle centred at $O$ and with radius $1$ choose the respective points $A_1, A_2, ..., A_{2n}$ , for $n \in N^*$. The lenght of the projection of the vector $\overrightarrow{u} = \overrightarrow{OA_1} + \overrightarrow{OA_2} + ... + \overrightarrow{OA_{2n}}$ on the diameter is an odd integer. Show that the projection of that vector on the diameter is at least $1$.

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**4** Let $n \geq 2$ be a positive integer. Find the maximum number of segments with lenghts greater than $1$, determined by $n$ points which lie on a closed disc with radius $1$.

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