Danube Mathematical Competition 2009

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by LarrySnake, parmenides51, anonymouslonely

1 Let be $\triangle ABC$ .Let A', B', C' be the foot of perpendiculars from A, B and C respectively.
The points E and F are on the sides CB' and BC' respectively, such that $B'E \cdot C'F = BF \cdot CE$.
Show that $AEA'F$ is cyclic.

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2 Prove that all the positive integer numbers , except for the powers of 2, can be written as the sum of (at least two) consecutive natural numbers .

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3 Let n be a natural number. Determine the minimal number of equilateral triangles of side 1 to cover the surface of an equilateral triangle of side $n + \frac{1}{2n}$.

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4 Let be a, b, c positive integers.Prove that $|a - b\sqrt{c}| < \frac{1}{2b}$ is true if and only if $|a^2 - b^2c| < \sqrt{c}$.

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5 Let σ, τ be two permutations of the quantity $\{1, 2, ..., n\}$.
Prove that there is a function $f : \{1, 2, ..., n\} \to \{-1, 1\}$ such that for any $1 \le i \le j \le n$,
we have $\left|\sum_{k=i}^{j} f(\sigma(k))\right| \le 2$ and $\left|\sum_{k=i}^{j} f(\tau(k))\right| \le 2$
