Spain Mathematical Olympiad 2019

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–	Day 1
1	An integer set ***T*** is orensan if there exist integers **a¡b¡c**, where **a** and **c** are part of ***T***, but **b** is not part of ***T***. Count the number of subsets ***T*** of 1,2,...,2019 which are orensan.
2	Determine if there exists a finite set S formed by positive prime numbers so that for each integer $n \geq 2$, the number $2^2 + 3^2 + ... + n^2$ is a multiple of some element of S.
3	The real numbers a, b and c verify that the polynomial $p(x) = x^4 + ax^3 + bx^2 + ax + c$ has exactly three distinct real roots; these roots are equal to $\tan y$, $\tan 2y$ and $\tan 3y$, for some real number y. Find all possible values of y, $0 \leq y < \pi$.
–	Day 2
4	Find all pairs of integers (x, y) that satisfy the equation $3^4 2^3 (x^2 + y^2) = x^3 y^3$
5	We consider all pairs (x, y) of real numbers such that $0 \leq x \leq y \leq 1$.Let $M(x, y)$ the maximum value of the set $$A = \{xy, 1 - x - y + xy, x + y - 2xy\}.$$ Find the minimum value that $M(x, y)$ can take for all these pairs (x, y).
6	In the scalene triangle ABC, the bisector of angle A cuts side BC at point D. The tangent lines to the circumscribed circunferences of triangles ABD and ACD on point D, cut lines AC and AB on points E and F respectively. Let G be the intersection point of lines BE and CF. Prove that angles EDG and ADF are equal.