Bundeswettbewerb Mathematik 2019

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–	Round 1
1	An 8×8 chessboard is covered completely and without overlaps by 32 dominoes of size 1×2. Show that there are two dominoes forming a 2×2 square.
2	The lettes A, C, F, H, L and S represent six not necessarily distinct decimal digits so that $S \neq 0$ and $F \neq 0$. We form the two six-digit numbers $SCHLAF$ and $FLACHS$. Show that the difference of these two numbers is divisible by 271 if and only if $C = L$ and $H = A$. *Remark:* The words "Schlaf" and "Flachs" are German for "sleep" and "flax".
3	Let $ABCD$ be a square. Choose points E on BC and F on CD so that $\angle EAF = 45^\circ$ and so that neither E nor F is a vertex of the square. The lines AE and AF intersect the circumcircle of the square in the points G and H distinct from A, respectively. Show that the lines EF and GH are parallel.
4	In the decimal expansion of $\sqrt{2} = 1.4142\dots$, Isabelle finds a sequence of k successive zeroes where k is a positive integer. Show that the first zero of this sequence can occur no earlier than at the k-th position after the decimal point.
–	Round 2
1	120 pirates distribute 119 gold pieces among themselves. Then the captain checks if any pirate has 15 or more gold pieces. If he finds the first one, he must give all his gold pieces to other pirates, whereby he may not give more than one gold piece to anyone. This control is repeated as long as there is any pirate with 15 or more gold pieces. Does this process end after a lot of checks?
2	Determine the smallest possible value of the sum $S(a, b, c) = \frac{ab}{c} + \frac{bc}{a} + \frac{ca}{b}$ where a, b, c are three positive real numbers with $a^2 + b^2 + c^2 = 1$
3	Let ABC be atriangle with $\overline{AC} > \overline{BC}$ and incircle k. Let M, W, L be the intersections of the median, angle bisector and altitude from point C respectively. The tangent to k passing through

M, that is different from AB, touch k in T. Prove that the angles $\angle MTW$ and $\angle TLM$ are equal.

4 Prove that for no integer $k \geq 2$, between $10k$ and $10k + 100$ there are more than 23 prime numbers.