Moscow Mathematical Olympiad 1950

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–	tour 1
173	On a chess board, the boundaries of the squares are assumed to be black. Draw a circle of the greatest possible radius lying entirely on the black squares.
174	a) Given 555 weights: of 1 g, 2 g, 3 g, . . . , 555 g, divide them into three piles of equal mass. b) Arrange 81 weights of $1^2, 2^2, ..., 81^2$ (all in grams) into three piles of equal mass.
175	a) We are given n circles $O_1, O_2, ..., O_n$, passing through one point O. Let $A_1, ..., A_n$ denote the second intersection points of O_1 with O_2, O_2 with O_3, etc., O_n with O_1, respectively. We choose an arbitrary point B_1 on O_1 and draw a line segment through A_1 and B_1 to the second intersection with O_2 at B_2, then draw a line segment through A_2 and B_2 to the second intersection with O_3 at B_3, etc., until we get a point B_n on O_n. We draw the line segment through B_n and A_n to the second intersection with O_1 at B_{n+1}. If B_k and A_k coincide for some k, we draw the tangent to O_k through A_k until this tangent intersects O_{k+1} at B_{k+1}. Prove that B_{n+1} coincides with B_1. b) for $n = 3$ the same problem
176	Let a, b, c be the lengths of the sides of a triangle and A, B, C, the opposite angles. Prove that $Aa + Bb + Cc \ge \frac{Ab+Ac+Ba+Bc+Ca+Cb}{2}$.
177	In a country, one can get from some point A to any other point either by walking, or by calling a cab, waiting for it, and then being driven. Every citizen always chooses the method of transportation that requires the least time. It turns out that the distances and the traveling times are as follows: 1 km takes 10 min, 2 km takes 15 min, 3 km takes 17.5 min. We assume that the speeds of the pedestrian and the cab, and the time spent waiting for cabs, are all constants. How long does it take to reach a point which is 6 km from A?
178	Let A be an arbitrary angle,let B and C be acute angles. Is there an angle x such that $\sin x = \frac{\sin B \cdot \sin C}{1-\cos B \cdot \cos C \cdot \cos A}$?
179	Two triangular pyramids have common base. One pyramid contains the other. Can the sum of the lengths of the edges of the inner pyramid be longer than that of the outer one?
180	Solve the equation $\sqrt{x+3-4\sqrt{x-1}}+\sqrt{x+8-6\sqrt{x-1}}=1$.
–	tour 2

181 a) In a convex 13-gon all diagonals are drawn, dividing it into smaller polygons. What is the greatest number of sides can these polygons have?
b) In a convex 1950-gon all diagonals are drawn, dividing it into smaller polygons. What is the greatest number of sides can these polygons have?

182 Prove that $\frac{1}{2}\frac{3}{4}\frac{5}{6}\frac{7}{8}...\frac{99}{100} < \frac{1}{10}$.

183 A circle is inscribed in a triangle and a square is circumscribed around this circle so that no side of the square is parallel to any side of the triangle. Prove that less than half of the squares perimeter lies outside the triangle.

184 * On a circle, 20 points are chosen. Ten non-intersecting chords without mutual endpoints connect some of the points chosen. How many distinct such arrangements are there?

185 The numbers $1, 2, 3, ..., 101$ are written in a row in some order. Prove that it is always possible to erase 90 of the numbers so that the remaining 11 numbers remain arranged in either increasing or decreasing order.

186 A spatial quadrilateral is circumscribed around a sphere. Prove that all the tangent points lie in one plane.

187 Is it possible to draw 10 bus routes with stops such that for any 8 routes there is a stop that does not belong to any of the routes, but any 9 routes pass through all the stops?