Moscow Mathematical Olympiad 1951

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by parmenides51

– tour 1

188 Prove that $x^{12} - x^9 + x^4 - x + 1 > 0$ for all x.

189 Let $ABCD$ and $A'B'C'D'$ be two convex quadrilaterals whose corresponding sides are equal, i.e., $AB = A'B', BC = B'C'$, etc. Prove that if $\angle A > \angle A'$, then $\angle B < \angle B', \angle C > \angle C', \angle D < \angle D'$.

190 Which number is greater: $\frac{2.00000000004}{(1.00000000004)^2+2.00000000004}$ or $\frac{2.00000000002}{(1.00000000002)^2+2.00000000002}$?

191 Given an isosceles trapezoid $ABCD$ and a point P.
Prove that a quadrilateral can be constructed from segments PA, PB, PC, PD.

192 a) Given a chain of 60 links each weighing 1 g. Find the smallest number of links that need to be broken if we want to be able to get from the obtained parts all weights 1 g, 2 g, $\ldots$, 59 g, 60 g? A broken link also weighs 1 g.
b) Given a chain of 150 links each weighing 1 g. Find the smallest number of links that need to be broken if we want to be able to get from the obtained parts all weights 1 g, 2 g, $\ldots$, 149 g, 150 g? A broken link also weighs 1 g.

193 Prove that the first 3 digits after the decimal point in the decimal expression of the number $\frac{0.123456789101112...495051}{0.515049...121110987654321}$ are 239.

194 One side of a convex polygon is equal to a, the sum of exterior angles at the vertices not adjacent to this side are equal to 120^o. Among such polygons, find the polygon of the largest area.

195 We have two concentric circles. A polygon is circumscribed around the smaller circle and is contained entirely inside the greater circle. Perpendiculars from the common center of the circles to the sides of the polygon are extended till they intersect the greater circle. Each of the points obtained is connected with the endpoints of the corresponding side of the polygon . When is the resulting star-shaped polygon the unfolding of a pyramid?

196 Given three equidistant parallel lines. Express by points of the corresponding lines the values of the resistance, voltage and current in a conductor so as to obtain the voltage $V = I \cdot R$ by connecting with a ruler the points denoting the resistance R and the current I. (Each point of each scale denotes only one number).

Three parallel straight lines are given at equal distances from each other. How to depict by points of the corresponding straight lines the values of resistance, voltage and the current in the conductor, so that, applying a ruler to to points depicting the values of resistance R and values of current I, obtain on the voltage scale a point depicting the value of voltage V = I R (point each scale represents one and only one number).

– tour 2

197 Prove that the number $1\underbrace{0...0}_{49}5\underbrace{0...0}_{99}1$ is not the cube of any integer.

198 * On a plane, given points A, B, C and angles $\angle D, \angle E, \angle F$ each less than 180^o and the sum equal to 360^o, construct with the help of ruler and protractor a point O such that $\angle AOB = \angle D, \angle BOC = \angle E$ and $\angle COA = \angle F$.

199 Prove that the sum $1^3 + 2^3 + ... + n^3$ is a perfect square for all n.

200 What figure can the central projection of a triangle be?
(The center of the projection does not lie on the plane of the triangle.)

201 To prepare for an Olympiad 20 students went to a coach. The coach gave them 20 problems and it turned out that
(a) each of the students solved two problems and
(b) each problem was solved by twostudents.
Prove that it is possible to organize the coaching so that each student would discuss one of the problems that (s)he had solved, and so that all problems would be discussed.

202 Dividing $x^{1951} - 1$ by $P(x) = x^4 + x^3 + 2x^2 + x + 1$ one gets a quotient and a remainder.
Find the coefficient of x^{14} in the quotient.

203 A sphere is inscribed in an n-angled pyramid. Prove that if we align all side faces of the pyramid with the base plane, flipping them around the corresponding edges of the base, then
(1) all tangent points of these faces to the sphere would coincide with one point, H, and
(2) the vertices of the faces would lie on a circle centered at H.

204 * Given several numbers each of which is less than 1951 and the least common multiple of any two of which is greater than 1951. Prove that the sum of their reciprocals is less than 2.

205 Among all orthogonal projections of a regular tetrahedron to all possible planes, find the projection of the greatest area.

206 Consider a curve with the following property:
inside the curve one can move an inscribed equilateral triangle so that each vertex of the triangle

moves along the curve and draws the whole curve.
Clearly, every circle possesses the property.
Find a closed planar curve without self-intersections, that has the property but is not a circle.

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207 * A bus route has 14 stops (counting the first and the last). A bus cannot carry more than 25 passengers. We assume that a passenger takes a bus from A to B if (s)he enters the bus at A and gets off at B.
Prove that for any bus route:
a) there are 8 distinct stops $A_1, B_1, A_2, B_2, A_3, B_3, A_4, B_4$ such that no passenger rides from A_k to B_k for all $k = 1, 2, 3, 4$ #)
b) there might not exist 10 distinct stops $A_1, B_1, ..., A_5, B_5$ with property #).
