

2019 Israel National Olympiad

www.artofproblemsolving.com/community/c919189

by Cuubic

- 1 In kindergarden, there are 32 children and three classes: Judo, Agriculture, and Math. Every child is in exactly one class and every class has at least one participant.

One day, the teacher gathered 6 children to clean up the classroom. The teacher counted and found that exactly $\frac{1}{2}$ of the Judo students, $\frac{1}{4}$ of the Agriculture students and $\frac{1}{8}$ of the Math students are cleaning.

How many children are in each class?

- 2 We are given a 5×5 square grid, divided to 1×1 tiles. Two tiles are called **linked** if they lie in the same row or column, and the distance between their centers is 2 or 3. For example, in the picture the gray tiles are the ones linked to the red tile.

<https://i.imgur.com/JVTQ9wB.png>

Sammy wants to mark as many tiles in the grid as possible, such that no two of them are linked. What is the maximal number of tiles he can mark?

- 3 Six congruent isosceles triangles have been put together as described in the picture below. Prove that points M, F, C lie on one line.

<https://i.imgur.com/1LU5Zmb.png>

- 4 In the beginning, the number 1 is written on the board 9999 times. We are allowed to perform the following actions:

- Erase four numbers of the form x, x, y, y , and instead write the two numbers $x + y, x - y$. (The order or location of the erased numbers does not matter)

- Erase the number 0 from the board, if it's there.

Is it possible to reach a state where:

- Only one number remains on the board?

- At most three numbers remain on the board?

- 5 Guy has 17 cards. Each of them has an integer written on it (the numbers are not necessarily positive, and not necessarily different from each other). Guy noticed that for each card, the square of the number written on it equals the sum of the numbers on the 16 other cards.

What are the numbers on Guy's cards? Find all of the options.

- 6 A set of integers is called **legendary** if you can reach any integer from it by using the following action multiple times:

If the numbers x, y are in the set, we may add the number $xy - y^2 - y + x$ to the set.

Prove that any legendary set contains at least 8 numbers.

- 7 In the plane points A, B, C are marked in blue and points P, Q are marked in red (no 3 marked points lie on a line, and no 4 marked points lie on a circle). A circle is called **separating** if all points of one color are inside it, and all points of the other color are outside of it. Denote by O the circumcenter of ABC and by R the circumradius of ABC .

Prove that **exactly one** of the following holds:

- There exists a separating circle;
 - There exists a point X on the segment PQ which also lies inside the triangle ABC , for which $PX \cdot XQ = R^2 - OX^2$.
-