2014 Israel National Olympiad

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by Cuubic

1 Consider the number $(101^2-100^2)\cdot(102^2-101^2)\cdot(103^2-102^2)\cdot...\cdot(200^2-199^2)$.

- Determine its units digit.
- Determine its tens digit.

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2 Let $\Delta A_1A_2A_3,\Delta B_1B_2B_3,\Delta C_1C_2C_3$ be three equilateral triangles. The vertices in each triangle are numbered clockwise. It is given that $A_3=B_3=C_3$. Let M be the center of mass of $\Delta A_1B_1C_1$, and let N be the center of mass of $\Delta A_2B_2C_2$.

Prove that ΔA_3MN is an equilateral triangle.

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3 Let $ABCDEF$ be a convex hexagon. In the hexagon there is a point K, such that $ABCK,DEFK$ are both parallelograms. Prove that the three lines connecting A,B,C to the midpoints of segments CE,DF,EA meet at one point.

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4 We are given a row of $n\geq7$ tiles. In the leftmost 3 tiles, there is a white piece each, and in the rightmost 3 tiles, there is a black piece each. The white and black players play in turns (the white starts). In each move, a player may take a piece of their color, and move it to an adjacent tile, so long as it's not occupied by a piece of the same color. If the new tile is empty, nothing happens. If the tile is occupied by a piece of the opposite color, both pieces are destroyed (both white and black). The player who destroys the last two pieces wins the game.

Which player has a winning strategy, and what is it? (The answer may depend on n)

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5 Let p be a polynomial with integer coefficients satisfying $p(16)=36,p(14)=16,p(5)=25$. Determine all possible values of $p(10)$.

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6 Let n be a positive integer. Find the maximal real number k, such that the following holds:

For any n real numbers $x_1,x_2,...,x_n$, we have $\sqrt{x_1^2+x_2^2+\dots+x_n^2}\geq k\cdot\min(|x_1-x_2|,|x_2-x_3|,...,|x_{n-1}-x_n|,|x_n-x_1|)$

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7 Find one real value of x satisfying $\frac{x^7}{7}=1+\sqrt[7]{10}x\left(x^2-\sqrt[7]{10}\right)^2$.
