2013 Israel National Olympiad

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by Cuubic

1 In the picture there are six coins, each with radius 1cm. Each coin is tangent to exactly two other coins next to it (as in the picture). Between the coins, there is an empty area whose boundary is a star-like shape. What is the perimeter of this shape?
`https://i.imgur.com/aguQRVd.png`

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2 Let $A = \{n \in \mathbb{Z} \mid 0 < n < 2013\}$. A subset $B \subseteq A$ is called **reduced** if for any two numbers $x, y \in B$, we must have $x \cdot y \notin B$. For example, any subset containing the numbers $3, 5, 15$ cannot be reduced, and same for a subset containing $4, 16$.

- Find the maximal size of a reduced subset of A.
- How many reduced subsets are there with that maximal size?

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3 Let $p(x) = x^4 - 5773x^3 - 46464x^2 - 5773x + 46$. Determine the sum of $\arctan$-s of its real roots.

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4 Determine the number of positive integers n satisfying:

- $n < 10^6$
- n is divisible by 7
- n does not contain any of the digits 2,3,4,5,6,7,8.

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5 A point in the plane is called **integral** if both its x and y coordinates are integers. We are given a triangle whose vertices are integral. Its sides do not contain any other integral points. Inside the triangle, there are exactly 4 integral points. Must those 4 points lie on one line?

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6 Let $x_1, ..., x_n$ be positive real numbers, satisfying $x_1 + \cdots + x_n = n$. Prove that

$$\frac{x_1}{x_2} + \frac{x_2}{x_3} + \cdots + \frac{x_{n-1}}{x_n} + \frac{x_n}{x_1} \leq \frac{4}{x_1 \cdot x_2 \cdot \cdots \cdot x_n} + n - 4.$$

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