2011 Israel National Olympiad

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by Cuubic

1 We are given 5771 weights weighing 1,2,3,...,5770,5771. We partition the weights into n sets of equal weight. What is the maximal n for which this is possible?

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2 Evaluate the sum $\sqrt{1-\frac{1}{2}\cdot\sqrt{1\cdot 3}}+\sqrt{2-\frac{1}{2}\cdot\sqrt{3\cdot 5}}+\sqrt{3-\frac{1}{2}\cdot\sqrt{5\cdot 7}}+\dots+\sqrt{40-\frac{1}{2}\cdot\sqrt{79\cdot 81}}$.

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3 In some foreign country's government, there are 12 ministers. Each minister has 5 friends and 6 enemies in the government (friendship/enemyship is a symmetric relation). A triplet of ministers is called **uniform** if all three of them are friends with each other, or all three of them are enemies. How many uniform triplets are there?

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4 Let $\alpha_1,\alpha_2,\alpha_3$ be three congruent circles that are tangent to each other. A third circle β is tangent to them at points A_1,A_2,A_3 respectively. Let P be a point on β which is different from A_1,A_2,A_3. For $i=1,2,3$, let B_i be the second intersection point of the line PA_i with circle α_i. Prove that $\Delta B_1B_2B_3$ is equilateral.

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5 We have two lists of numbers: One initially containing 1,6,11,...,96, and the other initially containing 4,9,14,...,99. In every turn, we erase two numbers from one of the lists, and write $\frac{1}{3}$ of their sum (not necessarily an integer) in the other list. We continue this process until there are no possible moves.

- Prove that at the end of the process, there is exactly one number in each list.
- Prove that those two numbers are not equal.

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6 There are N red cards and N blue cards. Each card has a positive integer between 1 and N (inclusive) written on it. Prove that we can choose a (non-empty) subset of the red cards and a (non-empty) subset of the blue cards, so that the sum of the numbers on the chosen red cards equals the sum of the numbers on the chosen blue cards.
