

China Second Round Olympiad 2001

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by Henry_2001, billzhao

- 1 Let O, H be the circumcenter and orthocenter of $\triangle ABC$, respectively. Line AH and BC intersect at D , Line BH and AC intersect at E , Line CH and AB intersect at F , Line AB and ED intersect at M , AC and FD intersect at N . Prove that (1) $OB \perp DF, OC \perp DE$; (2) $OH \perp MN$.

- 2 If nonnegative reals x_1, x_2, \dots, x_n satisfy

$$\sum_{i=1}^n x_i^2 + 2 \sum_{1 \leq k < j \leq n} \sqrt{\frac{k}{j}} x_k x_j = 1$$

what are the minimum and maximum values of $\sum_{i=1}^n x_i$?

- 3 An $m \times n$ ($m, n \in \mathbb{N}^*$) rectangle is divided into some smaller squares. The sides of each square are all parallel to the corresponding sides of the rectangle, and the length of each side is integer. Determine the minimum of the sum of the sides of these squares.