China Second Round Olympiad 2000

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1 In acute-angled triangle ABC, E, F are on the side BC, such that $\angle BAE = \angle CAF$, and let M, N be the projections of F onto AB, AC, respectively. The line AE intersects $\odot(ABC)$ at D(different from point A).
Prove that $S_{AMDN} = S_{\triangle ABC}$.

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2 Define the sequence $a_1, a_2, \ldots$ and $b_1, b_2, \ldots$ as $a_0 = 1, a_1 = 4, a_2 = 49$ and for $n \geq 0$

$$\begin{cases} a_{n+1} = 7a_n + 6b_n - 3, \\ b_{n+1} = 8a_n + 7b_n - 4. \end{cases}$$

Prove that for any non-negative integer n, a_n is a perfect square.

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3 There are n people, and given that any 2 of them have contacted with each other at most once. In any group of $n-2$ of them, any one person of the group has contacted with other people in this group for 3^k times, where k is a non-negative integer. Determine all the possible value of n.
