National Mathematical Olympiad 2009

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–	2nd Round
1	let O be the center of the circle inscribed in a rhombus ABCD. points E,F,G,H are chosen on sides AB, BC, CD, DA respectively so that EF and GH are tangent to inscribed circle. show that EH and FG are parallel.
2	a palindromic number is a number which is unchanged when order of its digits is reversed. prove that the arithmetic progression 18, 37,.. contains infinitely many palindromic numbers.
3	for $k \in \mathbb{N}$, define A_n for $n = 1, 2, ...$ by $A_{n+1} = \frac{nA_n + 2(n+1)^{2k}}{n+2}, A_1 = 1$ Prove A_n is integer for all $n \geq 1$, and A_n is odd if and only if $n \equiv$1 or 2(mod 4)
4	find largest constant C st $\sum_{i=1}^4 (x_i + 1/x_i)^3 \geq C$ for all positive real numbers $x_1, .., x_4$ st $x_1^3 + x_3^3 + 3x_1x_3 = x_2 + x_4 = 1$
5	Find all integers x,y,z with $2 \leq x \leq y \leq z$ st $xy \equiv 1$(mod z) $xz \equiv 1$(mod y) $yz \equiv 1$ (mod x)