National Mathematical Olympiad 2013

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–	2nd Round
1	Let a_1, a_2, ... be a sequence of integers defined recursively by $a_1 = 2013$ and for $n \geq 1$, a_{n+1} is the sum of the 2013-th powers of the digits of a_n. Do there exist distinct positive integers i, j such that $a_i = a_j$?
2	Let ABC be an acute-angled triangle and let D, E, and F be the midpoints of BC, CA, and AB respectively. Construct a circle, centered at the orthocenter of triangle ABC, such that triangle ABC lies in the interior of the circle. Extend EF to intersect the circle at P, FD to intersect the circle at Q and DE to intersect the circle at R. Show that $AP = BQ = CR$.
3	Let n be a positve integer. prove there exists a positive integer n st $n^{2013} - n^{20} + n^{13} - 2013$ has at least N distinct prime factors.
4	Let F be a finite non-empty set of integers and let n be a positive integer. Suppose that • Any $x \in F$ may be written as $x = y + z$ for some $y, z \in F$; • If $1 \leq k \leq n$ and x_1, ..., $x_k \in F$, then $x_1 + \cdots + x_k \neq 0$. Show that F has at least $2n + 2$ elements.
5	Let ABC be a triangle with integral side lengths such that $\angle A = 3\angle B$. Find the minimum value of its perimeter.