National Mathematical Olympiad 2018

www.artofproblemsolving.com/community/c923879

by parmenides51, dominicleejun

– 2nd Round

1 Consider a regular cube with side length 2. Let A and B be 2 vertices that are furthest apart. Construct a sequence of points on the surface of the cube A_1, A_2, $\ldots$, A_k so that $A_1 = A$, $A_k = B$ and for any $i = 1, \ldots, k-1$, the distance from A_i to A_{i+1} is 3. Find the minimum value of k.

2 Let O be a point inside triangle ABC such that $\angle BOC$ is 90° and $\angle BAO = \angle BCO$. Prove that $\angle OMN$ is 90 degrees, where M and N are the midpoints of $\overline{AC}$ and $\overline{BC}$, respectively.

3 Let n be a positive integer. Show that there exists an integer m such that

$$2018m^2 + 20182017m + 2017$$

is divisible by 2^n.

4 each of the squares in a 2 x 2018 grid of squares is to be coloured black or white such that in any 2 x 2 block , at least one of the 4 squares is white. let P be the number of ways of colouring the grid. find the largest k so that 3^k divides P.

5 Consider a polynomial $P(x,y,z)$ in three variables with integer coefficients such that for any real numbers a,b,c,

$$P(a,b,c) = 0 \Leftrightarrow a = b = c.$$

Find the largest integer r such that for all such polynomials $P(x,y,z)$ and integers m,n,

$$m^r \mid P(n, n+m, n+2m).$$

Proposed by Ma Zhao Yu