Tournament Of Towns 1980

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– Junior

(001) 1 On the circumference of a circle there are red and blue points. One may add a red point and change the colour of both its neighbours (to the other colour) or remove a red point and change the colour of both its previous neighbours. Initially there are two red points. Prove that there is no sequence of allowed operations which leads to the configuration consisting of two blue points.

(K Kazarnovskiy, Moscow)

(002) 2 In a $N \times N$ array of numbers, all rows are different (two rows are said to be different even if they differ only in one entry). Prove that there is a column which can be deleted in such a way that the resulting rows will still be different.

(A Andjans, Riga)

(003) 3 If permutations of the numbers $2, 3, 4, ..., 102$ are denoted by $a_i, a_2, a_3, ..., a_{101}$, find all such permutations in which a_k is divisible by k for all k.

(004) 4 We are given convex quadrilateral $ABCD$. Each of its sides is divided into N line segments of equal length. The points of division of side AB are connected with the points of division of side CD by straight lines (which we call the first set of straight lines), and the points of division of side BC are connected with the points of division of side DA by straight lines (which we call the second set of straight lines) as shown in the diagram, which illustrates the case $N = 4$. This forms N^2 smaller quadrilaterals. From these we choose N quadrilaterals in such a way that any two are at least divided by one line from the first set and one line from the second set. Prove that the sum of the areas of these chosen quadrilaterals is equal to the area of $ABCD$ divided by N.

(A Andjans, Riga)
`http://4.bp.blogspot.com/-8Qqk4r68nhU/XVco29-HzzI/AAAAAAAAKgo/UY8mXxg7tD0OrS6bEnoAw7Vuf3`
`s1600/TOT%2B1980%2BSpring%2BJ4.png`

(005) 5 A finite set of line segments, of total length 18, belongs to a square of unit side length (we assume that the square includes its boundary and that a line segment includes its end points). The line segments are parallel to the sides of the square and may intersect one another. Prove that among the regions into which the square is divided by the line segments, at least one of these must have area not less than 0.01.

(A Berzinsh, Riga)

–	Senior
1	same as Junior Q1
2	same as Junior Q2
(006) 3	We are given 30 non-zero vectors in 3 dimensional space. Prove that among these there are two such that the angle between them is less than 45^o.
4	same as Junior Q4
5	same as Junior Q5