Tournament Of Towns 1981

www.artofproblemsolving.com/community/c925475

by parmenides51

– Spring 1981

– Junior

(007) 1 Find all integer solutions to the equation $y^k = x^2 + x$, where k is a natural number greater than 1.

(008) 2 M is a finite set of points in a plane. Point O in the plane is called an almost centre of symmetry of set M if it is possible to remove from M one point in such a way that among the remaining members O is the centre of symmetry in the usual sense. How many such almost centres of symmetry may a finite point set in a plane have? Indicate all such points.

(V Prasolov, Moscow)

(009) 3 $ABCD$ is a convex quadrilateral inscribed in a circle with centre O, and with mutually perpendicular diagonals. Prove that the broken line AOC divides the quadrilateral into two parts of equal area.

(V Varvarkin)

(010) 4 Each of K friends simultaneously learns one different item of news. They begin to phone one another to tell them their news. Each conversation lasts exactly one hour, during which time it is possible for two friends to tell each other all of their news. What is the minimum number of hours needed in order for all of the friends to know all of the news? Consider in this problem
(a) $K = 64$.
(b) $K = 55$.
(c) $K = 100$.

(A Andjans, Riga)

PS. (a) was the junior problem, (a),(b),(c) the senior one

(011) 5 a) A game is played on an infinite plane. There are fifty one pieces, one wolf and 50 sheep. There are two players. The first commences by moving the wolf. Then the second player moves one of the sheep, the first player moves the wolf, the second player moves a sheep, and so on. The wolf and the sheep can move in any direction through a distance of up to one metre per move. Is it true that for any starting position the wolf will be able to capture at least one sheep?
b) A game is played on an infinite plane. There are two players. One has a piece known as a wolf, while the other has K pieces known as sheep. The first player moves the wolf, then the

second player moves a sheep, the first player moves the wolf again, the second player moves a sheep, and so on. The wolf and the sheep can move in any direction, with a maximum distance of one metre per move. Is it true that for any value of K there exists an initial position from which the wolf can not capture any sheep?

PS. (a) was the junior version, (b) the senior one

– Senior

---

(012) 1 We will say that two pyramids touch each other by faces if they have no common interior points and if the intersection of a face of one of them with a face of the other is either a triangle or a polygon. Is it possible to place 8 tetrahedra in such a way that every two of them touch each other by faces?

(A Andjans, Riga)

---

2 variation of Junior Q5

---

(013) 3 Prove that every real positive number may be represented as a sum of nine numbers whose decimal representation consists of the digits 0 and 7.

(E Turkevich)

---

4 variation of Junior Q4

---

(014) 5 On an infinite squared sheet six squares are shaded as in the diagram. On some squares there are pieces. It is possible to transform the positions of the pieces according to the following rule: if the neighbour squares to the right and above a given piece are free, it is possible to remove this piece and put pieces on these free squares.
The goal is to have all the shaded squares free of pieces. Is it possible to reach this goal if
(a) In the initial position there are 6 pieces and they are placed on the 6 shaded squares?
(b) In the initial position there is only one piece, located in the bottom left shaded square?
https://cdn.artofproblemsolving.com/attachments/2/d/0d5cbc159125e2a84edd6ac6aca5206bf8d83
png
(M Kontsevich, Moscow)
