China Second Round Olympiad 1999

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1 In convex quadrilateral $ABCD, \angle BAC = \angle CAD$. E lies on segment CD, and BE and AC intersect at F, DF and BC intersect at G. Prove that $\angle GAC = \angle EAC$.

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2 Let a,b,c be real numbers.
Let z_1,z_2,z_3 be complex numbers such that $|z_k| = 1\ (k = 1, 2, 3)$ and $\frac{z_1}{z_2} + \frac{z_2}{z_3} + \frac{z_3}{z_1} = 1$
Find $|az_1 + bz_2 + cz_3|$.

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3 n is a given positive integer, such that its possible to weigh out the mass of any product weighing $1, 2, 3, \cdots, ng$ with a counter balance without sliding poise and k counterweights, which weigh $x_i g(i = 1, 2, \cdots, k)$, respectively, where $x_i \in \mathbb{N}^*$ for any $i \in \{1, 2, \cdots, k\}$ and $x_1 \leq x_2 \leq \cdots \leq x_k$.

(1)Let $f(n)$ be the least possible number of k. Find $f(n)$ in terms of n. (2)Find all possible number of n, such that sequence $x_1, x_2, \cdots, x_{f(n)}$ is uniquely determined.
