AoPS Online
Art of Problem Solving



**geometry problems from Olimpiada de Mayo, level 2, max 15 years old**

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– level 2

**1995.4** Consider a pyramid whose base is an equilateral triangle $BCD$ and whose other faces are triangles isosceles, right at the common vertex $A$. An ant leaves the vertex $B$ arrives at a point $P$ of the $CD$ edge, from there goes to a point $Q$ of the edge $AC$ and returns to point $B$. If the path you made is minimal, how much is the angle $PQA$ ?

**1996.1** Let $ABCD$ be a rectangle. A line $r$ moves parallel to $AB$ and intersects diagonal $AC$ , forming two triangles opposite the vertex, inside the rectangle. Prove that the sum of the areas of these triangles is minimal when $r$ passes through the midpoint of segment $AD$ .

**1996.4** Let $ABCD$ be a square and let point $F$ be any point on side $BC$. Let the line perpendicular to $DF$, that passes through $B$, intersect line $DC$ at $Q$. What is value of $\angle FQC$?

**1997.2** In a square $ABCD$ with side $k$, let $P$ and $Q$ in $BC$ and $DC$ respectively, where $PC = 3PB$ and $QD = 2QC$. Let $M$ be the point of intersection of the lines $AQ$ and $PD$, determine the area of $QMD$ in function of $k$

**1997.5** What are the possible areas of a hexagon with all angles equal and sides $1, 2, 3, 4, 5$, and $6$, in some order?

**1998.2** Let $ABC$ be an equilateral triangle. $N$ is a point on the side $AC$ such that $\vec{AC} = 7\vec{AN}$, $M$ is a point on the side $AB$ such that $MN$ is parallel to $BC$ and $P$ is a point on the side $BC$ such that $MP$ is parallel to $AC$. Find the ratio of areas $\frac{(MNP)}{(ABC)}$

**1999.2** In a unit circle where $O$ is your circumcenter, let $A$ and $B$ points in the circle with $\angle BOA = 90$. In the arc $AB$(minor arc) we have the points $P$ and $Q$ such that $PQ$ is parallel to $AB$. Let $X$ and $Y$ be the points of intersections of the line $PQ$ with $OA$ and $OB$ respectively. Find the value of $PX^2 + PY^2$

**1999.4** Let $ABC$ be an equilateral triangle. $M$ is the midpoint of segment $AB$ and $N$ is the midpoint of segment $BC$. Let $P$ be the point outside $ABC$ such that the triangle $ACP$ is isosceles and right in $P$. $PM$ and $AN$ are cut in $I$. Prove that $CI$ is the bisector of the angle $MCA$ .

**1999.5** There are $12$ points that are vertices of a regular polygon with $12$ sides. Rafael must draw segments that have their two ends at two of the points drawn. He is allowed to have each point be an endpoint of more than one segment and for the segments to intersect, but he is prohibited from drawing three segments that are the three sides of a triangle in which each vertex is one of the $12$ starting points. Find the maximum number of segments Rafael can draw and justify why he cannot draw a greater number of segments.

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**2000.2** Given a parallelogram with area $1$ and we will construct lines where this lines connect a vertex with a midpoint of the side no adjacent to this vertex; with the $8$ lines formed we have a octagon inside of the parallelogram. Determine the area of this octagon

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**2000.3** Let $S$ be a circle with radius $2$, let $S_1$ be a circle,with radius $1$ and tangent, internally to $S$ in $B$ and let $S_2$ be a circle, with radius $1$ and tangent to $S_1$ in $A$, but $S_2$ isn't tangent to $S$. If $K$ is the point of intersection of the line $AB$ and the circle $S$, prove that $K$ is in the circle $S_2$.

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**2001.2** On the trapezoid $ABCD$ , side $DA$ is perpendicular to the bases $AB$ and $CD$ . The base $AB$ measures $45$, the base $CD$ measures $20$ and the $BC$ side measures $65$. Let $P$ on the $BC$ side such that $BP$ measures $45$ and $M$ is the midpoint of $DA$. Calculate the measure of the $PM$ segment.

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**2001.4** Ten coins of $1$ cm radius are placed around a circle as indicated in the figure.
Each coin is tangent to the circle and its two neighboring coins.
Prove that the sum of the areas of the ten coins is twice the area of the circle.
`https://cdn.artofproblemsolving.com/attachments/5/e/edf7a7d39d749748f4ae818853cb3f8b2b35b`
`gif`

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**2002.3** In a triangle $ABC$, right in $A$ and isosceles, let $D$ be a point on the side $AC$ ($A \neq D \neq C$) and $E$ be the point on the extension of $BA$ such that the triangle $ADE$ is isosceles. Let $P$ be the midpoint of segment $BD$, $R$ be the midpoint of the segment $CE$ and $Q$ the intersection point of $ED$ and $BC$. Prove that the quadrilateral $ARQP$ is a square

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**2003.2** Let $ABCD$ be a rectangle of sides $AB = 4$ and $BC = 3$. The perpendicular on the diagonal $BD$ drawn from $A$ cuts $BD$ at point $H$. We call $M$ the midpoint of $BH$ and $N$ the midpoint of $CD$. Calculate the measure of the segment $MN$.

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**2003.4** Bob plotted $2003$ green points on the plane, so all triangles with three green vertices have area less than $1$.
Prove that the $2003$ green points are contained in a triangle $T$ of area less than $4$.

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**2003.5** An ant, which is on an edge of a cube of side $8$, must travel on the surface and return to the starting point. It's path must contain interior points of the six faces of the cube and should visit only once each face of the cube. Find the length of the path that the ant can carry out and justify why it is the shortest path.

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**2004.3** We have a pool table $8$ meters long and $2$ meters wide with a single ball in the center. We throw the ball in a straight line and, after traveling $29$ meters, it stops at a corner of the table. How many times did the ball hit the edges of the table?

Note: When the ball rebounds on the edge of the table, the two angles that form its trajectory with the edge of the table are the same.

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**2005.1** The enemy ship has landed on a $9 \times 9$ board that covers exactly $5$ squares of the board, like this:
https://cdn.artofproblemsolving.com/attachments/2/4/ae5aa95f5bb5e113fd5e25931a2bf8eb872db
png
The ship is invisible. Each defensive missile covers exactly one square, and destroys the ship if it hits one of the $5$ squares that it occupies. Determine the minimum number of missiles needed to destroy the enemy ship with certainty .

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**2005.3** In a triangle $ABC$ with $AB = AC$, let $M$ be the midpoint of $CB$ and let $D$ be a point in $BC$ such that $\angle BAD = \frac{\angle BAC}{6}$. The perpendicular line to $AD$ by $C$ intersects $AD$ in $N$ where $DN = DM$. Find the angles of the triangle $BAC$.

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**2006.4** Let $ABCD$ be a trapezoid of bases $AB$ and $CD$ . Let $O$ be the intersection point of the diagonals $AC$ and $BD$. If the area of the triangle $ABC$ is $150$ and the area of the triangle $ACD$ is $120$, calculate the area of the triangle $BCO$.

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**2007.5** In the triangle $ABC$ we have $\angle A = 2\angle C$ and $2\angle B = \angle A + \angle C$. The angle bisector of $\angle C$ intersects the segment $AB$ in $E$, let $F$ be the midpoint of $AE$, let $AD$ be the altitude of the triangle $ABC$. The perpendicular bisector of $DF$ intersects $AC$ in $M$.
Prove that $AM = CM$.

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**2008.2** Let $ABCD$ be a rectangle and $P$ be a point on the side$AD$ such that $\angle BPC = 90^o$. The perpendicular from $A$ on $BP$ cuts $BP$ at $M$ and the perpendicular from $D$ on $CP$ cuts $CP$ in $N$. Show that the center of the rectangle lies in the $MN$ segment.

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**2008.4** In the plane we have $16$ lines(not parallel and not concurrents), we have $120$ point(s) of intersections of this lines.
Sebastian has to paint this $120$ points such that in each line all the painted points are with colour differents, find the minimum(quantity) of colour(s) that Sebastian needs to paint this points.
If we have have $15$ lines(in this situation we have $105$ points), what's the minimum(quantity) of colour(s)?

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**2009.2** Let $ABCD$ be a convex quadrilateral such that the triangle $ABD$ is equilateral and the triangle $BCD$ is isosceles, with $\angle C = 90^o$. If $E$ is the midpoint of the side $AD$, determine the measure of the angle $\angle CED$.

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**2010.2** Let $ABCD$ be a rectangle and the circle of center $D$ and radius $DA$, which cuts the extension of the side $AD$ at point $P$. Line $PC$ cuts the circle at point $Q$ and the extension of the side $AB$ at point $R$. Show that $QB = BR$.

 

**2011.3** In a right triangle rectangle $ABC$ such that $AB = AC$, $M$ is the midpoint of $BC$. Let $P$ be a point on the perpendicular bisector of $AC$, lying in the semi-plane determined by $BC$ that does not contain $A$. Lines $CP$ and $AM$ intersect at $Q$. Calculate the angles that form the lines $AP$ and $BQ$.

**2012.3** Given Triangle $ABC$, $\angle B = 2\angle C$, and $\angle A > 90^\circ$. Let $M$ be midpoint of $BC$. Perpendicular of $AC$ at $C$ intersects $AB$ at $D$. Show $\angle AMB = \angle DMC$

If possible, don't use projective geometry

**2012.4** Six points are given so that there are not three on the same line and that the lengths of the segments determined by these points are all different. We consider all the triangles that they have their vertices at these points. Show that there is a segment that is both the shortest side of one of those triangles and the longest side of another.

**2013.2** Construct the midpoint of a segment using an unmarked ruler and a *trisector* that marks in a segment the two points that divide the segment in three equal parts.

**2013.3** Many distinct points are marked in the plane. A student draws all the segments determined by those points, and then draws a line *r* that does not pass through any of the marked points, but cuts exactly $60$ drawn segments. How many segments were not cut by *r*? Give all possibilites.

**2014.2** In a convex quadrilateral $ABCD$, let $M$, $N$, $P$, and $Q$ be the midpoints of $AB$, $BC$, $CD$, and $DA$ respectively. If $MP$ and $NQ$ divide $ABCD$ in four quadrilaterals with the same area, prove that $ABCD$ is a parallelogram.

**2015.3** Let $ABCDEFGHI$ be a regular polygon of $9$ sides. The segments $AE$ and $DF$ intersect at $P$. Prove that $PG$ and $AF$ are perpendicular.

**2015.5** If you have $65$ points in a plane, we will make the lines that passes by any two points in this plane and we obtain exactly $2015$ distinct lines, prove that least $4$ points are collinears!!

**2016.4** In a triangle $ABC$, let $D$ and $E$ be points of the sides $BC$ and $AC$ respectively. Segments $AD$ and $BE$ intersect at $O$. Suppose that the line connecting midpoints of the triangle and parallel to $AB$, bisects the segment $DE$. Prove that the triangle $ABO$ and the quadrilateral $ODCE$ have equal areas.

**2016.5** Rosa and Sara play with a triangle $ABC$, right at $B$. Rosa begins by marking two interior points of the hypotenuse $AC$, then Sara marks an interior point of the hypotenuse $AC$ different from those of Rosa. Then, from these three points the perpendiculars to the sides $AB$ and $BC$ are drawn, forming the following figure.
`https://cdn.artofproblemsolving.com/attachments/9/9/c964bbacc4a5960bee170865cc43902410e5`

 

png
Sara wins if the area of the shaded surface is equal to the area of the unshaded surface, in other case wins Rosa. Determine who of the two has a winning strategy.

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**2017.3** Let $ABCD$ be a quadrilateral such that $\angle ABC = \angle ADC = 90$ and $\angle BCD$ ¿ $90$. Let $P$ be a point inside of the $ABCD$ such that $BCDP$ is parallelogram, the line $AP$ intersects $BC$ in $M$. If $BM = 2, MC = 5, CD = 3$. Find the length of $AM$.

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**2018.4** In a parallelogram $ABCD$, let $M$ be the point on the $BC$ side such that $MC = 2BM$ and let $N$ be the point of side $CD$ such that $NC = 2DN$. If the distance from point $B$ to the line $AM$ is $3$, calculate the distance from point $N$ to the line $AM$.

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**2018.5** Each point on a circle is colored with one of $10$ colors. Is it true that for any coloring there are $4$ points of the same color that are vertices of a quadrilateral with two parallel sides (an isosceles trapezoid or a rectangle)?

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**2019.3** On the sides $AB, BC$ and $CA$ of a triangle $ABC$ are located the points $P, Q$ and $R$ respectively, such that $BQ = 2QC, CR = 2RA$ and $\angle PRQ = 90^o$. Show that $\angle APR = \angle RPQ$.

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**2019.5** We consider the $n$ vertices of a regular polygon with $n$ sides. There is a set of triangles with vertices at these $n$ points with the property that for each triangle in the set, the sides of at least one are not the side of any other triangle in the set. What is the largest amount of triangles that can have the set?

Consideramos los n vértices de un polígono regular de n lados. Se tiene un conjunto de triángulos con vértices en estos n puntos con la propiedad que para cada triángulo del conjunto, al menos uno
de sus lados no es lado de ningún otro triángulo del conjunto. ¿Cuál es la mayor cantidad de triángulos que puede tener el conjunto?

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**2020.4** Let $ABC$ be a right triangle, right at $B$, and let $M$ be the midpoint of the side $BC$. Let $P$ be the point in
bisector of the angle $\angle BAC$ such that $PM$ is perpendicular to $BC (P$ is outside the triangle $ABC$). Determine the triangle area $ABC$ if $PM = 1$ and $MC = 5$.

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**2021.3** Let $ABC$ be a triangle and $D$ is a point inside of the triangle, such that $\angle DBC = 60^\circ$ and $\angle DCB = \angle DAB = 30^\circ$. Let $M$ and $N$ be the midpoints of $AC$ and $BC$, respectively. Prove that $\angle DMN = 90^\circ$.

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**2022.3** Let $ABCD$ be a square, $E$ a point on the side $CD$, and $F$ a point inside the square such that that triangle $BFE$ is isosceles and $\angle BFE = 90^o$ . If $DF = DE$, find the measure of angle $\angle FDE$.

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**2022.5** The vertices of a regular polygon with $N$ sides are marked on the blackboard. Ana and Beto

play alternately, Ana begins. Each player, in turn, must do the following: • join two vertices with a segment, without cutting another already marked segment; or • delete a vertex that does not belong to any marked segment.
The player who cannot take any action on his turn loses the game. Determine which of the two players can guarantee victory:
a) if $N = 28$
b) if $N = 29$