Dutch BxMO/EGMO Team Selection Test 2014

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by 61plus

1 Find all non-negative integer numbers n for which there exists integers a and b such that $n^2 = a + b$ and $n^3 = a^2 + b^2$.

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2 Find all functions $f : \mathbb{R}\backslash\{0\} \to \mathbb{R}$ for which $xf(xy) + f(-y) = xf(x)$ for all non-zero real numbers x, y.

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3 In triangle ABC, I is the centre of the incircle. There is a circle tangent to AI at I which passes through B. This circle intersects AB once more in P and intersects BC once more in Q. The line QI intersects AC in R. Prove that $|AR| \cdot |BQ| = |PI|^2$

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4 Let $m \geq 3$ and n be positive integers such that $n > m(m-2)$. Find the largest positive integer d such that $d \mid n!$ and $k \nmid d$ for all $k \in \{m, m+1, \ldots, n\}$.

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5 Let n be a positive integer. Daniel and Merlijn are playing a game. Daniel has k sheets of paper lying next to each other on a table, where k is a positive integer. On each of the sheets, he writes some of the numbers from 1 up to n (he is allowed to write no number at all, or all numbers). On the back of each of the sheets, he writes down the remaining numbers. Once Daniel is nished, Merlijn can ip some of the sheets of paper (he is allowed to ip no sheet at all, or all sheets). If Merlijn succeeds in making all of the numbers from 1 up to n visible at least once, then he wins. Determine the smallest k for which Merlijn can always win, regardless of Daniels actions.
