Gulf Mathematical Olympiad 2016

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by m2121

1 Consider sequences a_0,a_1,a_2,$\cdots$ of non-negative integers defined by selecting any a_0,a_1,a_2 (not all 0) and for each $n \geq 3$ letting a_n = —$a_n - 1$ - $a_n - 3$—

1-In the particular case that a_0 = 1,a_1 = 3 and a_2 = 2, calculate the beginning of the sequence, listing a_0,a_1,$\cdots$,a_{19},a_{20}.

2-Prove that for each sequence, there is a constant c such that $a_i \leq c$ for all $i \geq 0$. Note that the constant c my depend on the numbers a_0,a_1 and a_2

3-Prove that, for each choice of a_0,a_1 and a_2, the resulting sequence is eventually periodic.

4-Prove that, the minimum length p of the period described in (3) is the same for all permitted starting values a_0,a_1,a_2 of the sequence

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2 Let x be a real number that satisfies $x^1 + x^{-1} = 3$
Prove that $x^n + x^{-n}$ is an positive integer , then prove that the positive integer $x^{3^{1437}} + x^{3^{-1437}}$ is divisible by at least 1439×2^{1437} positive integers

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3 Consider the acute-angled triangle ABC. Let X be a point on the side BC, and Y be a point on the side CA. The circle k_1 with diameter AX cuts AC again at E' .The circle k_2 with diameter BY cuts BC again at B'.
(i) Let M be the midpoint of XY . Prove that $A'M = B'M$.
(ii) Suppose that k_1 and k_2 meet at P and Q. Prove that the orthocentre of ABC lies on the line PQ.

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4 4. Suppose that four people A, B, C and D decide to play games of tennis doubles. They might first play the team A and B against the team C and D. Next A and C might play B and D. Finally A and D might play B and C. The advantage of this arrangement is that two conditions are satisfied.
(a) Each player is on the same team as each other player exactly once.
(b) Each player is on the opposing team to each other player exactly twice.
Is it possible to arrange a collection of tennis matches satisfying both condition (a) and condition (b) in the following circumstances?
(i) There are five players.
(ii) There are seven players.
(iii) There are nine players.
