Gulf Mathematical Olympiad 2017

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1 1- Find a pair (m, n) of positive integers such that $K = |2^m - 3^n|$ in all of this cases :

$a)K = 5$ $b)K = 11$ $c)K = 19$

2-Is there a pair (m, n) of positive integers such that :

$$|2^m - 3^n| = 2017$$

3-Every prime number less than 41 can be represented in the form $|2^m - 3^n|$ by taking an Appropriate pair (m, n)
of positive integers. Prove that the number 41 cannot be represented in the form $|2^m - 3^n|$ where m and n are positive integers

4-Note that $2^5 + 3^2 = 41$. The number 53 is the least prime number that cannot be represented as a sum or an difference of a power of 2 and a power of 3 . Prove that the number 53 cannot be represented in any of the forms $2^m - 3^n$, $3^n - 2^m$, $2^m - 3^n$ where m and n are positive integers

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2 One country consists of islands $A_1, A_2, \cdots, A_N$,The ministry of transport decided to build some bridges such that anyone will can travel by car from any of the islands $A_1, A_2, \cdots, A_N$ to any another island by one or more of these bridges. For technical reasons the only bridges that can be built is between A_i and A_{i+1} where $i = 1, 2, \cdots, N - 1$, and between A_i and A_N where $i < N$.

We say that a plan to build some bridges is good if it is satisfies the above conditions , but when we remove any bridge it will not satisfy this conditions. We assume that there is a_N of good plans. Observe that $a_1 = 1$ (The only good plan is to not build any bridge) , and $a_2 = 1$ (We build one bridge).

1-Prove that $a_3 = 3$
2-Draw at least 5 different good plans in the case that $N = 4$ and the islands are the vertices of a square
3-Compute a_4
4-Compute a_6
5-Prove that there is a positive integer i such that 1438 divides a_i

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3

Let C_1 and C_2 be two different circles , and let their radii be r_1 and r_2 , the two circles are passing through the two points A and B
(i)Let P_1 on C_1 and P_2 on C_2 such that the line P_1P_2 passes through A. Prove that $P_1B \cdot r_2 = P_2B \cdot r_1$

(ii)Let DEF be a triangle that it's inscribed in C_1 , and let $D'E'F'$ be a triangle that is inscribed in C_2 . The lines EE',DD' and FF' all pass through A . Prove that the triangles DEF and $D'E'F'$ are similar

(iii)The circle C_3 also passes through A and B . Let l be a line that passes through A and cuts circles C_i in M_i with $i = 1, 2, 3$. Prove that the value of

$$\frac{M_1M_2}{M_1M_3}$$

is constant regardless of the position of l Provided that l is different from AB

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4 1 - Prove that $55 < (1+\sqrt{3})^4 < 56$.

2 - Find the largest power of 2 that divides $\lceil (1+\sqrt{3})^{2n} \rceil$ for the positive integer n

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