Dutch BxMO/EGMO Team Selection Test 2013

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1 In quadrilateral $ABCD$ the sides AB and CD are parallel. Let M be the midpoint of diagonal AC. Suppose that triangles ABM and ACD have equal area. Prove that $DM//BC$.

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2 Consider a triple (a, b, c) of pairwise distinct positive integers satisfying $a+b+c = 2013$. A step consists of replacing the triple (x, y, z) by the triple $(y+z-x, z+x-y, x+y-z)$. Prove that, starting from the given triple (a, b, c), after 10 steps we obtain a triple containing at least one negative number.

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3 Find all triples (x, n, p) of positive integers x and n and primes p for which the following holds $x^3 + 3x + 14 = 2p^n$

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4 Determine all functions $f : \mathbb{R} \to \mathbb{R}$ satisfying

$$f(x + yf(x)) = f(xf(y)) - x + f(y + f(x))$$

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5 Let $ABCD$ be a cyclic quadrilateral for which $|AD| = |BD|$. Let M be the intersection of AC and BD. Let I be the incentre of $\triangle BCM$. Let N be the second intersection pointof AC and the circumscribed circle of $\triangle BMI$. Prove that $|AN| \cdot |NC| = |CD| \cdot |BN|$.
