Dutch BxMO Team Selection Test 2016

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1 For a positive integer n that is not a power of two, we de fine $t(n)$ as the greatest odd divisor of n and $r(n)$ as the smallest positive odd divisor of n unequal to 1. Determine all positive integers n that are not a power of two and for which we have $n = 3t(n) + 5r(n)$.

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2 Determine all triples (x, y, z) of non-negative real numbers that satisfy the following system of equations $\begin{cases} x^2 - y = (z-1)^2 \\ y^2 - z = (x-1)^2 \\ z^2 - x = (y-1)^2 \end{cases}$.

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3 Let $\triangle ABC$ be a right-angled triangle with $\angle A = 90^o$ and circumcircle Γ. The inscribed circle is tangent to BC in point D. Let E be the midpoint of the arc AB of Γ not containing C and let F be the midpoint of the arc AC of Γ not containing B.
(a) Prove that $\triangle ABC \sim \triangle DEF$.
(b) Prove that EF goes through the points of tangency of the incircle to AB and AC.

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4 The Facebook group Olympiad training has at least five members. There is a certain integer k with following property: [i]for each k-tuple of members there is at least one member of this k-tuple friends with each of the other $k-1$.[/i]
(Friendship is mutual: if A is friends with B, then also B is friends with A.)
(a) Suppose $k = 4$. Can you say with certainty that the Facebook group has a member that is friends with each of the other members?
(b) Suppose $k = 5$. Can you say with certainty that the Facebook group has a member that is friends with each of the other members?

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5 Determine all pairs (m, n) of positive integers for which $(m+n)^3/2n(3m^2+n^2)+8$
