Dutch BxMO Team Selection Test 2018

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by parmenides51

1 We have 1000 balls in 40 different colours, 25 balls of each colour. Determine the smallest n for which the following holds: if you place the 1000 balls in a circle, in any arbitrary way, then there are always n adjacent balls which have at least 20 different colours.

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2 Let $\triangle ABC$ be a triangle of which the side lengths are positive integers which are pairwise coprime. The tangent in A to the circumcircle intersects line BC in D. Prove that BD is not an integer.

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3 Let p be a prime number.
Prove that it is possible to choose a permutation $a_1, a_2, ..., a_p$ of $1, 2, ..., p$ such that the numbers $a_1, a_1a_2, a_1a_2a_3, ..., a_1a_2a_3...a_p$ all have different remainder upon division by p.

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4 In a non-isosceles triangle $\triangle ABC$ we have $\angle BAC = 60^o$. Let D be the intersection of the angular bisector of $\angle BAC$ with side BC, O the centre of the circumcircle of $\triangle ABC$ and E the intersection of AO and BC. Prove that $\angle AED + \angle ADO = 90^o$.

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5 Let n be a positive integer. Determine all positive real numbers x satisfying $nx^2 + \frac{2^2}{x+1} + \frac{3^2}{x+2} + ... + \frac{(n+1)^2}{x+n} = nx + \frac{n(n+3)}{2}$
