4th Bay Area Mathematical Olympiad 2002

www.artofproblemsolving.com/community/c933594

by parmenides51, Plops

– February 26, 2002

---

1 Let ABC be a right triangle with right angle at B. Let $ACDE$ be a square drawn exterior to triangle ABC. If M is the center of this square, find the measure of $\angle MBC$.

---

2 In the illustration, a regular hexagon and a regular octagon have been tiled with rhombuses. In each case, the sides of the rhombuses are the same length as the sides of the regular polygon.
(a) Tile a regular decagon (10-gon) into rhombuses in this manner.
(b) Tile a regular dodecagon (12-gon) into rhombuses in this manner.
(c) How many rhombuses are in a tiling by rhombuses of a 2002-gon?
Justify your answer.
`https://cdn.artofproblemsolving.com/attachments/8/a/8413e4e2712609eba07786e34ba2ce4aa7288`
`png`

---

3 A game is played with two players and an initial stack of n pennies $(n \geq 3)$. The players take turns choosing one of the stacks of pennies on the table and splitting it into two stacks. The winner is the player who makes a move that causes all stacks to be of height 1 or 2. For which starting values of n does the player who goes first win, assuming best play by both players?

---

4 For $n \geq 1$, let a_n be the largest odd divisor of n, and let $b_n = a_1 + a_2 + ... + a_n$.
Prove that $b_n \geq \frac{n^2+2}{3}$, and determine for which n equality holds.

For example, $a_1 = 1, a_2 = 1, a_3 = 3, a_4 = 1, a_5 = 5, a_6 = 3$, thus $b_6 = 1 + 1 + 3 + 1 + 5 + 3 = 14 \geq \frac{6^2+2}{3} = 12\frac{2}{3}$
.

---

5 Professor Moriarty has designed a prime-testing trail. The trail has 2002 stations, labeled $1, ..., 2002$. Each station is colored either red or green, and contains a table which indicates, for each of the digits $0, ..., 9$, another station number. A student is given a positive integer n, and then walks along the trail, starting at station 1. The student reads the first (leftmost) digit of n, and looks this digit up in station 1s table to get a new station location. The student then walks to this new station, reads the second digit of n and looks it up in this stations table to get yet another station location, and so on, until the last (rightmost) digit of n has been read and looked up, sending the student to his or her final station. Here is an example that shows possible values for some of the tables. Suppose that $n = 19$:
`https://cdn.artofproblemsolving.com/attachments/f/3/db47f6761ca1f350e39d53407a1250c92c4b0`
`png`

Using these tables, station 1, digit 1 leads to station 29m station 29, digit 9 leads to station 1429, and
station 1429 is green.
Professor Moriarty claims that for any positive integer n, the final station (in the example, 1429) will be green if and only if n is prime. Is this possible?