Cono Sur Olympiad 2019

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by parmenides51, Jafet98

–	Day 1
1	Martin has two boxes A and B. In the box A there are 100 red balls numbered from 1 to 100, each one with one of these numbers. In the box B there are 100 blue balls numbered from 101 to 200, each one with one of these numbers. Martin chooses two positive integers a and b, both less than or equal to 100, and then he takes out a balls from box A and b balls from box B, without replacement. Martin's goal is to have two red balls and one blue ball among all balls taken such that the sum of the numbers of two red balls equals the number of the blue ball. What is the least possible value of $a+b$ so that Martin achieves his goal for sure? For such a minimum value of $a+b$, give an example of a and b satisfying the goal and explain why every a and b with smaller sum cannot accomplish the aim.
2	We say that a positive integer M with $2n$ digits is *hypersquared* if the following three conditions are met: -M is a perfect square. -The number formed by the first n digits of M is a perfect square. -The number formed by the last n digits of M is a perfect square and has exactly n digits (its first digit is not zero). Find a hypersquared number with 2000 digits.
3	Let $n \geq 3$ an integer. Determine whether there exist permutations $(a_1, a_2, \ldots, a_n)$ of the numbers $(1, 2, \ldots, n)$ and $(b_1, b_2, \ldots, b_n)$ of the numbers $(n+1, n+2, \ldots, 2n)$ so that $(a_1b_1, a_2b_2, \ldots a_nb_n)$ is a strictly increasing arithmetic progression.
–	Day 2
4	Find all positive prime numbers p, q, r, s so that $p^2 + 2019 = 26(q^2 + r^2 + s^2)$.
5	Let $n \geq 3$ a positive integer. In each cell of a $n \times n$ chessboard one must write 1 or 2 in such a way the sum of all written numbers in each 2×3 and 3×2 sub-chessboard is even. How many different ways can the chessboard be completed?
6	Let ABC be an acute-angled triangle with $AB < AC$, and let H be its orthocenter. The circumference with diameter AH meets the circumscribed circumference of ABC at $P \neq A$. The

tangent to the circumscribed circumference of ABC through P intersects line BC at Q. Show that $QP = QH$.