Argentina Iberoamerican Team Selection Test 2008

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–	Day 1
1	We have 100 equal cubes. Player A has to paint the faces of the cubes, each white or black, such that every cube has at least one face of each colour, at least 50 cubes have more than one black face and at least 50 cubes have more than one white face . Player B has to place the coloured cubes in a table in a way that their bases form the frame that surrounds a $40 * 12$ rectangle. There are some faces that can not been seen because they are overlapped with other faces or based on the table, we call them invisible faces. On the other hand, the ones which can be seen are called visible faces. Prove that player B can always place the cubes in such a way that the number of visible faces is the the same as the number of invisible faces, despite the initial colouring of player A Note: It is easy to see that in the configuration, each cube has three visible faces and three invisible faces
2	Two circunmferences Γ_1 Γ_2 intersect at A and B r_1 is the tangent from A to Γ_1 and r_2 is the tangent from B to Γ_2 $r_1 \cap r_2 = C$ $T = r_1 \cap \Gamma_2$ ($T \neq A$) We consider a point X in Γ_1 which is distinct from A and B. $XA \cap \Gamma_2 = Y$ ($Y \neq A$) $YB \cap XC = Z$ Prove that $TZ \parallel XY$
3	Show that exists a sequence of 100 terms such that: 1)Every term is a perfect square 2) every term is greater than the one before it (it is strictly increasing) 3)Every two terms of the sequence are relative prime 4) The average between two consecutive terms is also a perfect square Daniel
–	Day 2
1	Find all integers x such that $x(x+1)(x+7)(x+8)$ is a perfect square It's a nice problem ...hope you enjoy it!

Daniel

2 Set $S = \{1, 2, 3, ..., 2005\}$. If among any n pairwise coprime numbers in S there exists at least a prime number, find the minimum of n.

3 The plane is divided into regions by $n \geq 3$ lines, no two of which are parallel, and no three of which are concurrent. Some regions are coloured , in such a way that no two coloured regions share a common segment or half-line of their borders. Prove that the number of coloured regions is at most $\frac{n(n+1)}{3}$