Argentina Iberoamerican Team Selection Test 2009

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– Day 1

1 In the vertexes of a regular 31-gon there are written the numbers from 1 to 31, ordered increasingly, clockwise oriented.
We are allowed to perform an operation which consists in taking any three vertexes, namely the ones who have written a,b, and c and change them into c, $a-\frac{1}{10}$ and $b+\frac{1}{10}$ respectively (a becomes c, b becomes $a-\frac{1}{10}$ and c turns into $b+\frac{1}{10}$
Prove that after applying several operations we can reach the state in which the numbers in the vertexes are the numbers from 1 to 31, ordered increasingly,anti-clockwise oriented.

2 There are $m+1$ horizontal lines and m vertical lines on the plane so that $m(m+1)$ intersections are made.
A mark is placed at one of the m points of the lowest horizontal line.
2 players play the game of the following rules on this lines and points.

1. Each player moves a mark from a point to a point along the lines in turns.
2. The segment is erased after a mark moved along it.
3. When a player cannot make a move, then he loses.

Prove that the lead always wins the game.

PS I haven't found a student who solved it. There can be no one.

3 Let ABC be an isosceles triangle with $AC=BC$. Its incircle touches AB in D and BC in E. A line distinct of AE goes through A and intersects the incircle in F and G. Line AB intersects line EF and EG in K and L, respectively. Prove that $DK=DL$.

– Day 2

1 Find all positive integers (x,y) such that $\frac{y^2x}{x+y}$ is a prime number

2 Let a and k be positive integers. Let a_i be the sequence defined by $a_1=a$ and $a_{n+1}=a_n+k\pi(a_n)$
where $\pi(x)$ is the product of the digits of x (written in base ten)
Prove that we can choose a and k such that the infinite sequence a_i contains exactly 100 distinct terms

3 Within a group of 2009 people, every two people has exactly one common friend. Find the least value of the difference between the person with maximum number of friends and the person with minimum number of friends.