Federal Competition For Advanced Students, Part 1, 2009

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1 Show that for all positive integer n the following inequality holds $3^{n^2} > (n!)^4$
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2 For a positive integers n, k we define k-multifactorial of n as $Fk(n)$ = (n) . $(n-k)$ $(n-2k)$...(r), where r is the reminder when n is divided by k that satisfy $1 <= r <= k$
Determine all non-negative integers n such that $F20(n) + 2009$ is a perfect square.

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3 There are n bus stops placed around the circular lake. Each bus stop is connected by a road to the two adjacent stops (we call a *segment* the entire road between two stops). Determine the number of bus routes that start and end in the fixed bus stop A, pass through each bus stop at least once and travel through exactly $n+1$ *segments*.

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4 Let D, E, and F be respectively the midpoints of the sides BC, CA, and AB of $\triangle\ ABC$. Let H_a, H_b, H_c be the feet of perpendiculars from A, B, C to the opposite sides, respectively. Let P, Q, R be the midpoints of the H_bH_c, H_cH_a, and H_aH_b respectively. Prove that PD, QE, and RF are concurrent.
