Regional Competition For Advanced Students 2014

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1 Show that there are no positive real numbers x, y, z such $(12x^2+yz)(12y^2+xz)(12z^2+xy) = 2014x^2y^2z^2$.

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2 You can determine all 4-ples (a, b, c, d) of real numbers, which solve the following equation system $\begin{cases} ab+ac=3b+3c \\ bc+bd=5c+5d \\ ac+cd=7a+7d \\ ad+bd=9a+9b \end{cases}$

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3 The sequence (a_n) is defined with the recursion $a_{n+1}=5a_n^6+3a_{n-1}^3+a_{n-2}^2$ for $n \geq 2$ and the set of initial values $\{a_0, a_1, a_2\} = \{2013, 2014, 2015\}$. (That is, the initial values are these three numbers in any order.)
Show that the sequence contains no sixth power of a natural number.

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4 For a point P in the interior of a triangle ABC let D be the intersection of AP with BC, let E be the intersection of BP with AC and let F be the intersection of CP with AB.Furthermore let Q and R be the intersections of the parallel to AB through P with the sides AC and BC, respectively. Likewise, let S and T be the intersections of the
parallel to BC through P with the sides AB and AC, respectively.In a given triangle ABC, determine all points P for which the triangles PRD, $PEQ$and PTE have the same area.
