Czech And Slovak Mathematical Olympiad, Round III, Category A 2014

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by byk7, parmenides51

1 Let be n a positive integer. Denote all its (positive) divisors as $1 = d_1 < d_2 < \cdots < d_{k-1} < d_k = n$.
Find all values of n satisfying $d_5 - d_3 = 50$ and $11d_5 + 8d_7 = 3n$.

(Day 1, 1st problem
author: Mat Harminc)

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2 A segment AB is given in (Euclidean) plane. Consider all triangles XYZ such, that X is an inner point of AB, triangles XBY and XZA are similar (in this order of vertices) and points A, B, Y, Z lie on a circle in this order. Find a set of midpoints of all such segments YZ.

(Day 1, 2nd problem
authors: Michal Rolnek, Jaroslav vrek)

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3 Suppose we have a 8×8 chessboard. Each edge have a number, corresponding to number of possibilities of dividing this chessboard into 1×2 domino pieces, such that this edge is part of this division. Find out the last digit of the sum of all these numbers.

(Day 1, 3rd problem
author: Michal Rolnek)

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4 234 viewers came to the cinema. Determine for which$n \geq 4$ the viewers could be can be arranged in n rows so that every viewer in i-th row gets to know just j viewers in j-th row for any $i, j \in \{1, 2, ..., n\}, i \neq j$. (The relationship of acquaintance is mutual.)

(Tom Jurk)

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5 Given is the acute triangle ABC. Let us denote k a circle with diameter AB. Another circle, tangent to AB at point A and passing through point C intersects the circle k at point $P, P \neq A$. Another circle which touches AB at point B and passes point C, intersects the circle k at point $Q, Q \neq B$. Prove that the intersection of the line AQ and BP lies on one of the sides of angle ACB.

(Peter Novotn)

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6 For arbitrary non-negative numbers a and b prove inequality $\frac{a}{\sqrt{b^2+1}} + \frac{b}{\sqrt{a^2+1}} \geq \frac{a+b}{\sqrt{ab+1}}$, and find, where equality occurs.

(Day 2, 6th problem
authors: Tom Jurk, Jaromr ima)

