Czech And Slovak Mathematical Olympiad, Round III, Category A 2006

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by littletush

1 Define a sequence of positive integers $\{a_n\}$ through the recursive formula: $a_{n+1} = a_n + b_n (n \geq 1)$,where b_n is obtained by rearranging the digits of a_n (in decimal representation) in reverse order (for example,if $a_1 = 250$,then $b_1 = 52, a_2 = 302$,and so on). Can a_7 be a prime?

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2 Let m, n be positive integers such that the equation (in respect of x)

$$(x+m)(x+n) = x+m+n$$

has at least one integer root. Prove that $\frac{1}{2}n < m < 2n$.

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3 In a scalene triangle ABC,the bisectors of angle A, B intersect their corresponding sides at K, L respectively.I, O, H denote respectively the incenter,circumcenter and orthocenter of triangle ABC. Prove that A, B, K, L, O are concyclic iff KL is the common tangent line of the circumcircles of the three triangles ALI, BHI and BKI.

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4 Given a segment AB in the plane. Let C be another point in the same plane,H, I, G denote the orthocenter,incenter and centroid of triangle ABC. Find the locus of M for which A, B, H, I are concyclic.

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5 Find all triples (p, q, r) of pairwise distinct primes such that

$$p \mid q+r, q \mid r+2p, r \mid p+3q.$$

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6 Find all real solutions (x, y, z) of the system of equations:

$$\begin{cases} \tan^2 x + 2\cot^2 2y = 1 \\ \tan^2 y + 2\cot^2 2z = 1 \\ \tan^2 z + 2\cot^2 2x = 1 \end{cases}$$
