Finnish National High School Mathematics Competition 2014

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by parmenides51

1 Determine the value of the expression $x^2+y^2+z^2$,
if $x+y+z=13$, $xyz=72$ and $\frac{1}{x}+\frac{1}{y}+\frac{1}{z}=\frac{3}{4}$.

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2 The center of the circumcircle of the acute triangle ABC is M, and the circumcircle of ABM meets BC and AC at P and Q ($P \neq B$). Show that the extension of the line segment CM is perpendicular to PQ.

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3 The points $P=(a,b)$ and $Q=(c,d)$ are in the first quadrant of the xy plane, and a,b,c and d are integers satisfying $a<b, a<c, b<d$ and $c<d$. A route from point P to point Q is a broken line consisting of unit steps in the directions of the positive coordinate axes. An allowed route is a route not touching the line $x=y$. Tetermine the number of allowed routes.

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4 The radius r of a circle with center at the origin is an odd integer.
There is a point (p^m, q^n) on the circle, with p,q prime numbers and m,n positive integers.
Determine r.

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5 Determine the smallest number $n \in Z_+$, which can be written as $n=\Sigma_{a\in A}a^2$, where A is a finite set of positive integers and $\Sigma_{a\in A}a=2014$.
In other words: what is the smallest positive number which can be written as a sum of squares of different positive integers summing to 2014?
