Finnish National High School Mathematics Competition 2015

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by parmenides51

1   Solve the equation $\sqrt{1+\sqrt{1+x}} = \sqrt[3]{x}$ for $x \geq 0$.

2   The lateral edges of a right square pyramid are of length a.
Let $ABCD$ be the base of the pyramid, E its top vertex and F the midpoint of CE.
Assuming that BDF is an equilateral triangle, compute the volume of the pyramid.

3   Determine the largest integer k for which 12^k is a factor of $120!$

4   Let n be a positive integer. Every square in a $n \times n$-square grid is either white or black.
How many such colourings exist, if every 2×2-square consists of exactly two white and two black squares?
The squares in the grid are identified as e.g. in a chessboard, so in general colourings obtained from each other by rotation are different.

5   Mikko takes a multiple choice test with ten questions. His only goal is to pass the test, and this requires seven points. A correct answer is worth one point, and answering wrong results in the deduction of one point. Mikko knows for sure that he knows the correct answer in the six first questions. For the rest, he estimates that he can give the correct answer to each problem with probability $p, 0 < p < 1$. How many questions Mikko should try?