Dutch Mathematical Olympiad 2007

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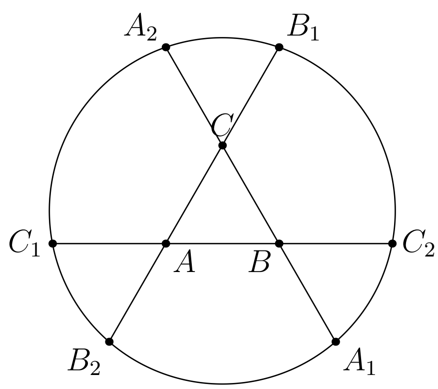
by parmenides51

1 Consider the equilateral triangle ABC with $|BC| = |CA| = |AB| = 1$.
On the extension of side BC, we define points A_1 (on the same side as B) and A_2 (on the same side as C) such that $|A_1B| = |BC| = |CA_2| = 1$. Similarly, we define B_1 and B_2 on the extension of side CA such that $|B_1C| = |CA| = |AB_2| = 1$, and C_1 and C_2 on the extension of side AB such that $|C_1A| = |AB| = |BC_2| = 1$. Now the circumcentre of 4ABC is also the centre of the circle that passes through the points A_1, B_2, C_1, A_2, B_1 and C_2.
Calculate the radius of the circle through A_1, B_2, C_1, A_2, B_1 and C_2.



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2 Is it possible to partition the set $A = \{1, 2, 3, ..., 32, 33\}$ into eleven subsets that contain three integers each, such that for every one of these eleven subsets, one of the integers is equal to the sum of the other two? If so, give such a partition, if not, prove that such a partition cannot exist.

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3 Does there exist an integer having the form $444...4443$ (all fours, and ending with a three) that is divisible by 13?
If so, give an integer having that form that is divisible by 13, if not, prove that such an integer cannot exist.

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4 Determine the number of integers a satisfying $1 \leq a \leq 100$ such that a^a is a perfect square.
(And prove that your answer is correct.)

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5 A triangle ABC and a point P inside this triangle are given.
Define D, E and F as the midpoints of AP, BP and CP, respectively. Furthermore, let R be the intersection of AE and BD, S the intersection of BF and CE, and T the intersection of CD and

AF.
Prove that the area of hexagon $DRESFT$ is independent of the position of P inside the triangle.