2019 China Hong Kong Math Olympaid

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1   Given that a, b, and c are positive real numbers such that $ab + bc + ca \geq 1$, prove that

$$\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2} \geq \frac{\sqrt{3}}{abc}.$$

2   Find the number of nonnegative integers k, $0 \leq k \leq 2188$, and such that $\binom{2188}{k}$ is divisible by 2188.

3   The incircle of $\triangle ABC$, with incentre I, meets BC, CA, and AB at D, E, and F, respectively. The line EF cuts the lines BI, CI, BC, and DI at K, L, M, and Q, respectively. The line through the midpoint of CL and M meets CK at P.

(a) Determine $\angle BKC$.
(b) Show that the lines PQ and CL are parallel.

4   Find all integers $n \geq 3$ with the following property: there exist n distinct points on the plane such that each point is the circumcentre of a triangle formed by 3 of the points.