Niels Henrik Abels Math Contest (Norwegian Math Olympiad) Final Round 2019

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1 You have an $n \times n$ grid of empty squares. You place a cross in all the squares, one at a time. When you place a cross in an empty square, you receive $i + j$ points if there were i crosses in the same row and j crosses in the same column before you placed the new cross. Which are the possible total scores you can get?

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2 $find$ all non negative integers m, n such that $mn - 1$ divides $n^3 - 1$

3a Three circles are pairwise tangent, with none of them lying inside another.
The centres of the circles are the corners of a triangle with circumference 1.
What is the smallest possible value for the sum of the areas of the circles?

3b Find all real functions f defined on the real numbers except zero, satisfying $f(2019) = 1$ and $f(x)f(y) + f\left(\frac{2019}{x}\right) f\left(\frac{2019}{y}\right) = 2f(xy)$ for all $x, y \neq 0$

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4 The diagonals of a convex quadrilateral $ABCD$ intersect at E. The triangles ABE, BCE, CDE and DAE have centroids K, L, M and N, and orthocentres Q, R, S and T. Show that the quadrilaterals $QRST$ and $LMNK$ are similar.
