Niels Henrik Abels Math Contest (Norwegian Math Olympiad) Final Round 2018

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by parmenides51

1 For an odd number n, we write $n!! = n \cdot (n-2)...3 \cdot 1$.
How many different residues modulo 1000 do you get from $n!!$ for $n = 1, 3, 5, $?

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2 The circumcentre of a triangle ABC is called O.
The points A', B' and C' are the reflections of O in BC, CA, and AB, respectively.
Show that the three lines AA', BB', and CC' meet in a common point.

3a Find all polynomials P such that $P(x) + 3P(x+2) = 3P(x+1) + P(x+3)$ for all real numbers x.

3b Find all real functions f defined on the real numbers except zero, satisfying $f(2019) = 1$ and $f(x)f(y) + f\left(\frac{2019}{x}\right) f\left(\frac{2019}{y}\right) = 2f(xy)$ for all $x, y \neq 0$

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4 Find all polynomials P such that $P(x) + \binom{2018}{2}P(x+2) + ... + \binom{2018}{2106}P(x+2016) + P(x+2018) =$
$= \binom{2018}{1}P(x+1) + \binom{2018}{3}P(x+3) + ... + \binom{2018}{2105}P(x+2015) + \binom{2018}{2107}P(x+2017)$
for all real numbers x.
