Niels Henrik Abels Math Contest (Norwegian Math Olympiad) Final Round 2017

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by parmenides51

1a Find all functions $f : R \to R$ which satisfy $f(x)f(y) = f(xy) + xy$ for all $x, y \in R$.

1b Find all functions $f : R \to R$ which satisfy $f(x)f(y) = f(x + y) + xy$ for all $x, y \in R$.

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2 Let the sequence an be defined by $a_0 = 2, a_1 = 15$, and $a_{n+2} = 15a_{n+1} + 16a_n$ for $n \geq 0$.
Show that there are infinitely many integers k such that $269|a_k$.

3a Nils has a telephone number with eight different digits.
He has made 28 cards with statements of the type The digit a occurs earlier than the digit b in my telephone number one for each pair of digits appearing in his number.
How many cards can Nils show you without revealing his number?

3b In an infinite grid of regular triangles, Niels and Henrik are playing a game they made up.
Every other time, Niels picks a triangle and writes $\times$ in it, and every other time, Henrik picks a triangle where he writes a o. If one of the players gets four in a row in some direction (see figure), he wins the game.
Determine whether one of the players can force a victory.
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4 Let $a > 0$ and $0 < \alpha < \pi$ be given. Let ABC be a triangle with $BC = a$ and $\angle BAC = \alpha$, and call the cicumcentre O, and the orthocentre H. The point P lies on the ray from A through O. Let S be the mirror image of P through AC, and T the mirror image of P through AB. Assume that $SATH$ is cyclic. Show that the length AP depends only on a and α.
