

Niels Henrik Abels Math Contest (Norwegian Math Olympiad) Final Round 2014

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by parmenides51, hungkg

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- 1a** Assume that $x, y \geq 0$. Show that $x^2 + y^2 + 1 \leq \sqrt{(x^3 + y + 1)(y^3 + x + 1)}$.
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- 1b** Find all functions $f : \mathbb{R} - \{0\} \rightarrow \mathbb{R}$ which satisfy $(1 + y)f(x) - (1 + x)f(y) = yf(x/y) - xf(y/x)$ for all real $x, y \neq 0$, and which take the values $f(1) = 32$ and $f(-1) = -4$.
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- 2** The points P and Q lie on the sides BC and CD of the parallelogram $ABCD$ so that $BP = QD$. Show that the intersection point between the lines BQ and DP lies on the line bisecting $\angle BAD$.
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- 3a** A grasshopper is jumping about in a grid. From the point with coordinates (a, b) it can jump to either $(a + 1, b)$, $(a + 2, b)$, $(a + 1, b + 1)$, $(a, b + 2)$ or $(a, b + 1)$. In how many ways can it reach the line $x + y = 2014$? Where the grasshopper starts in $(0, 0)$.
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- 3b** Nine points are placed on a circle. Show that it is possible to colour the 36 chords connecting them using four colours so that for any set of four points, each of the four colours is used for at least one of the six chords connecting the given points
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- 4** Find all triples (a, b, c) of positive integers for which $\frac{32a+3b+48c}{4abc}$ is also an integer.
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