Niels Henrik Abels Math Contest (Norwegian Math Olympiad) Final Round 2013

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by parmenides51

1a Find all real numbers a such that the inequality $3x^2+y^2 \geq -ax(x+y)$ holds for all real numbers x and y.

1b The sequence $a_1, a_2, a_3, ...$ is defined so that $a_1 = 1$ and $a_{n+1} = \frac{a_1+a_2+...+a_n}{n} + 1$ for $n \geq 1$. Show that for every positive real number b we can find a_k so that $a_k < bk$.

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2 In a triangle T, all the angles are less than 90^o, and the longest side has length s. Show that for every point p in T we can pick a corner h in T such that the distance from p to h is less than or equal to $s/\sqrt{3}$.

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3 A prime number $p \geq 5$ is given. Write $\frac{1}{3} + \frac{2}{4} + ... + \frac{p-3}{p-1} = \frac{a}{b}$ for natural numbers a and b. Show that p divides a.

4a An ordered quadruple (P_1, P_2, P_3, P_4) of corners in a regular 2013-gon is called *crossing* if the four corners are all different, and the line segment from P_1 to P_2 intersects the line segment from P_3 to P_4. How many *crossing* quadruples are there in the 2013-gon?

4b A total of $a \cdot b \cdot c$ cubical boxes are joined together in a $a \times b \times c$ rectangular stack, where $a, b, c \geq 2$. A bee is found inside one of the boxes. It can fly from one box to another through a hole in the wall, but not through edges or corners. Also, it cannot fly outside the stack. For which triples (a, b, c) is it possible for the bee to fly through all of the boxes exactly once, and end up in the same box where it started?
