**Niels Henrik Abels Math Contest (Norwegian Math Olympiad) Final Round 2010**

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by parmenides51, enndb0x

**1a** The point $P$ lies on the edge $AB$ of a quadrilateral $ABCD$.
The angles $BAD, ABC$ and $CPD$ are right, and $AB = BC + AD$.
Show that $BC = BP$ or $AD = BP$.

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**1b** The edges of the square in the figure have length $1$.
Find the area of the marked region in terms of $a$, where $0 \leq a \leq 1$.
`https://cdn.artofproblemsolving.com/attachments/2/2/f2b6ca973f66c50e39124913b3acb56feff8b`
`png`

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**2a** Show that $\frac{x^2}{1-x} + \frac{(1-x)^2}{x} \geq 1$ for all real numbers $x$, where $0 < x < 1$

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**2b** Show that $abc \leq (ab + bc + ca)(a^2 + b^2 + c^2)^2$ for all positive real numbers $a, b$ and $c$ such that $a + b + c = 1$.

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**3** $a)$
There are $25$ participants in a mathematics contest having four problems. Each problem is considered solved or not solved (that is, partial solutions are not possible). Show that either there are four contestants having solved the same problems (or not having solved any of them), or two contestants, one of which has solved exactly the problems that the other did not solve.
$b)$
There are $k$ sport clubs for the students of a secondary school. The school has $100$ students, and for any selection of three of them, there exists a club having at least one of them, but not all, as a member. What is the least possible value of $k$?

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**4a** Find all positive integers $k$ and $\ell$ such that $k^2 - \ell^2 = 1005$.

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**4b** Let $n > 2$ be an integer. Show that it is possible to choose $n$ points in the plane, not all of them lying on the same line, such that the distance between any pair of points is an integer (that is, $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$ is an integer for all pairs $(x_1, y_1)$ and $(x_2, y_2)$ of points).

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