Niels Henrik Abels Math Contest (Norwegian Math Olympiad) Final Round 2009

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by parmenides51

1a	Show that there exist infinitely many integers that cannot be written as the difference between two perfect squares.
1b	Show that the sum of three consecutive perfect cubes can always be written as the difference between two perfect squares.
2	There are two letters in a language. Every word consists of seven letters, and two different words always have different letters on at least three places. a. Show that such a language cannot have more than 16 words. b. Can there be 16 words in the language?
3a	In the triangle ABC the edge BC has length a, the edge AC length b, and the edge AB length c. Extend all the edges at both ends by the length a from the vertex A, b from B, and c from C. Show that the six endpoints of the extended edges all lie on a common circle. https://cdn.artofproblemsolving.com/attachments/8/7/14c8c6a4090d4fade28893729a510d263e7ab.png
3b	Show for any positive integer n that there exists a circle in the plane such that there are exactly n grid points within the circle. (A grid point is a point having integer coordinates.)
4a	Show that $\left(\frac{2010}{2009}\right)^{2009} > 2$.
4b	Let $x = 1 - 2^{-2009}$. Show that $x + x^2 + x^4 + x^8 + ... + x^{2^m} < 2010$ for all positive integers m.