Niels Henrik Abels Math Contest (Norwegian Math Olympiad) Final Round 2008

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by parmenides51

1 Let $s(n) = \frac{1}{6}n^3 - \frac{1}{2}n^2 + \frac{1}{3}n$.
(a) Show that $s(n)$ is an integer whenever n is an integer.
(b) How many integers n with $0 < n \leq 2008$ are such that $s(n)$ is divisible by 4?

2a We wish to lay down boards on a floor with width B in the direction across the boards. We have n boards of width b, and B/b is an integer, and $nb \leq B$. There are enough boards to cover the floor, but the boards may have different lengths. Show that we can cut the boards in such a way that every board length on the floor has at most one join where two boards meet end to end.
`https://cdn.artofproblemsolving.com/attachments/f/f/24ce8ae05d85fd522da0e18c0bb8017ca3c8e png`

2b A and B play a game on a square board consisting of $n \times n$ white tiles, where $n \geq 2$. A moves first, and the players alternate taking turns. A move consists of picking a square consisting of 2×2 or 3×3 white tiles and colouring all these tiles black. The first player who cannot find any such squares has lost. Show that A can always win the game if A plays the game right.

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3 a) Let x and y be positive numbers such that $x + y = 2$.
Show that $\frac{1}{x} + \frac{1}{y} \leq \frac{1}{x^2} + \frac{1}{y^2}$

b) Let x, y and z be positive numbers such that $x + y + z = 2$.
Show that $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} + \frac{9}{4} \leq \frac{1}{x^2} + \frac{1}{y^2} + \frac{1}{z^2}$
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4a Three distinct points A, B, and C lie on a circle with centre at O.
The triangles AOB, BOC , and COA have equal area.
What are the possible magnitudes of the angles of the triangle ABC ?

4b A point D lies on the side BC , and a point E on the side AC , of the triangle ABC , and BD and AE have the same length. The line through the centres of the circumscribed circles of the triangles ADC and BEC crosses AC in K and BC in L. Show that KC and LC have the same length.
