Dutch Mathematical Olympiad 2011

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by parmenides51

1 Determine all triples of positive integers (a, b, n) that satisfy the following equation: $a! + b! = 2^n$

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2 Let ABC be a triangle.
Points P and Q lie on side BC and satisfy $|BP| = |PQ| = |QC| = \frac{1}{3}|BC|$.
Points R and S lie on side CA and satisfy $|CR| = |RS| = |SA| = 13|CA|$.
Finally, points T and U lie on side AB and satisfy $|AT| = |TU| = |UB| = \frac{1}{3}|AB|$.
Points P, Q, R, S, T and U turn out to lie on a common circle.
Prove that ABC is an equilateral triangle.

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3 In a tournament among six teams, every team plays against each other team exactly once. When a team wins, it receives 3 points and the losing team receives 0 points. If the game is a draw, the two teams receive 1 point each.
Can the final scores of the six teams be six consecutive numbers $a, a + 1, ..., a + 5$?
If so, determine all values of a for which this is possible.

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4 Determine all pairs of positive real numbers (a, b) with $a > b$ that satisfy the following equations: $a\sqrt{a} + b\sqrt{b} = 134$ and $a\sqrt{b} + b\sqrt{a} = 126$.

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5 The number devil has coloured the integer numbers: every integer is coloured either black or white.
The number 1 is coloured white. For every two white numbers a and b (a and b are allowed to be equal) the numbers $a - b$ and a+b have di fferent colours.
Prove that 2011 is coloured white.
