Dutch Mathematical Olympiad 2014

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1 Determine all triples (a, b, c), where a, b, and c are positive integers that satisfy

$a \leq b \leq c$ and $abc = 2(a + b + c)$.

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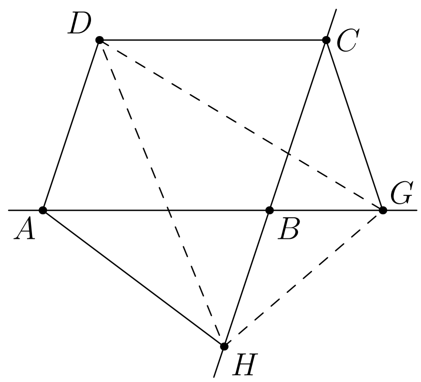
2 juniors Let $ABCD$ be a parallelogram with an acute angle at A. Let G be a point on the line AB, distinct from B, such that $|CG| = |CB|$. Let H be a point on the line BC, distinct from B, such that $|AB| = |AH|$. Prove that triangle DGH is isosceles.



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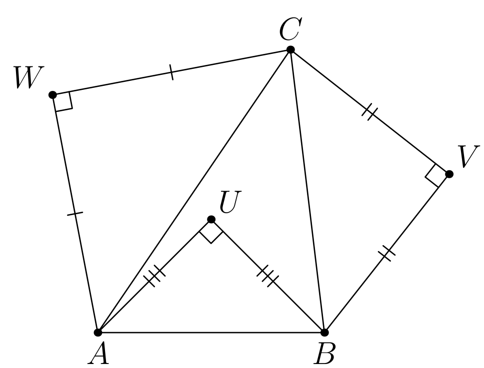
2 seniors On the sides of triangle ABC, isosceles right-angled triangles AUB, CVB, and AWC are placed. These three triangles have their right angles at vertices U, V , and W, respectively. Triangle AUB lies completely inside triangle ABC and triangles CVB and AWC lie completely outside ABC. See the figure. Prove that quadrilateral $UVCW$ is a parallelogram.



3 At a volleyball tournament, each team plays exactly once against each other team. Each game has a winning team, which gets 1 point. The losing team gets 0 points. Draws do not occur. In the nal ranking, only one team turns out to have the least number of points (so there is no shared last place). Moreover, each team, except for the team having the least number of points, lost exactly one game against a team that got less points in the final ranking.
a) Prove that the number of teams cannot be equal to 6.
b) Show, by providing an example, that the number of teams could be equal to 7.

4 A quadruple (p, a, b, c) of positive integers is called a Leiden quadruple if
- p is an odd prime number,
- a, b, and c are distinct and
- $ab + 1, bc + 1$ and $ca + 1$ are divisible by p.
a) Prove that for every Leiden quadruple (p, a, b, c) we have $p + 2 \leq \frac{a+b+c}{3}$.
b) Determine all numbers p for which a Leiden quadruple (p, a, b, c) exists with $p + 2 = \frac{a+b+c}{3}$

5 We consider the ways to divide a 1 by 1 square into rectangles (of which the sides are parallel to those of the square). All rectangles must have the same circumference, but not necessarily the same shape.
a) Is it possible to divide the square into 20 rectangles, each having a circumference of $2 : 5$?
b) Is it possible to divide the square into 30 rectangles, each having a circumference of 2?