Dutch Mathematical Olympiad 2015

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by parmenides51

1 We make groups of numbers. Each group consists of *fi ve* distinct numbers. A number may occur in multiple groups. For any two groups, there are exactly four numbers that occur in both groups.
(a) Determine whether it is possible to make 2015 groups.
(b) If all groups together must contain exactly *six* distinct numbers, what is the greatest number of groups that you can make?
(c) If all groups together must contain exactly *seven* distinct numbers, what is the greatest number of groups that you can make?

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2 On a 1000×1000-board we put dominoes, in such a way that each domino covers exactly two squares on the board. Moreover, two dominoes are not allowed to be adjacent, but are allowed to touch in a vertex.
Determine the maximum number of dominoes that we can put on the board in this way.

Attention: you have to really prove that a greater number of dominoes is impossible.

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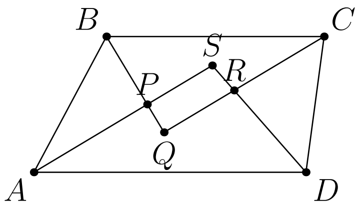
3 juniors In quadrilateral $ABCD$ sides BC and AD are parallel. In each of the four vertices we draw an angular bisector. The angular bisectors of angles A and B intersect in point P, those of angles B and C intersect in point Q, those of angles C and D intersect in point R, and those of angles D and A intersect in point S. Suppose that PS is parallel to QR. Prove that $|AB| = |CD|$.



Attention: the figure is not drawn to scale.

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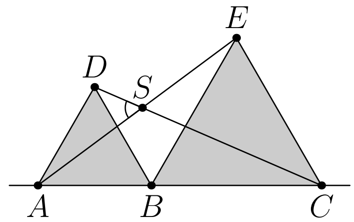
3 seniors Points A, B, and C are on a line in this order. Points D and E lie on the same side of this line, in such a way that triangles ABD and BCE are equilateral. The segments AE and CD intersect in point S. Prove that $\angle ASD = 60^o$.



4 Find all pairs of prime numbers (p, q) for which $7pq^2 + p = q^3 + 43p^3 + 1$

5 Given are (not necessarily positive) real numbers a, b, and c for which $|a - b| \geq |c|, |b - c| \geq |a|$ and $|c - a| \geq |b|$. Prove that one of the numbers a, b, and c is the sum of the other two.